

DURATION: 3 HRS.

MAX. MARKS:80

-) Question No. 1 is compulsory.
-) Attempt any **THREE** of the remaining.
-) Figures to the right indicate full marks.

1. A) Determine the constants a, b, c, d, e if

$$f(z) = (ax^4 + bx^2y^2 + cy^4 + dx^2 - 2y^2) + i(4x^3y - cxy^3 + 4xy) \text{ is analytic.} \quad (5)$$

B) Find half range Fourier sine series for $f(x) = x^2, 0 < x < 3.$ (5)

C) Find the directional derivative of $\varphi(x, y, z) = xy^2 + yz^3$ at the point (2, -1, 1) in the direction of the vector $i + 2j + 2k.$ (5)

D) Evaluate $\int_0^\infty e^{-2t} t^5 \cosh t \, dt.$ (5)

2. A) Prove that $J_{\frac{3}{2}}(x) = \sqrt{\frac{2}{\pi x}} \left(\frac{\sin x}{x} - \cos x \right)$ (6)

B) If $f(z) = u + iv$ is analytic and $u - v = e^x(\cos y - \sin y)$, find $f(z)$ in terms of $z.$ (6)

C) Obtain Fourier series for $f(x) = x + \frac{\pi}{2} \quad -\pi < x < 0$
 $= \frac{\pi}{2} - x \quad 0 < x < \pi$

Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$ (8)

3. A) Show that $\vec{F} = (2xy + z^3)i + x^2j + 3xz^2k$, is a conservative field. Find its scalar potential and also find the work done by the force \vec{F} in moving a particle from (1, -2, 1) to (3, 1, 4). (6)

B) Show that the set of functions $\{\sin(2n + 1)x\}, n = 0, 1, 2, \dots$ is orthogonal over $[0, \pi/2]$. Hence construct orthonormal set of functions. (6)

[TURN OVER]

C) Find (i) $L^{-1}\{\cot^{-1}(s+1)\}$

(ii) $L^{-1}\left(\frac{e^{-2s}}{s^2+8s+25}\right)$ (8)

Q.4) A) Prove that $\int J_3(x) dx = -\frac{2J_1(x)}{x} - J_2(x)$ (6)

B) Find inverse Laplace of $\frac{s}{(s^2+a^2)(s^2+b^2)}$ ($a \neq b$) using Convolution theorem. (6)

C) Expand $f(x) = x \sin x$ in the interval $0 \leq x \leq 2\pi$ as a Fourier series.

Hence, deduce that $\sum_{n=2}^{\infty} \frac{1}{n^2-1} = \frac{3}{4}$ (8)

Q.5) A) Using Gauss Divergence theorem evaluate $\iint_S \vec{N} \cdot \vec{F} ds$ where $\vec{F} = x^2\vec{i} + z\vec{j} + yz\vec{k}$

and S is the cube bounded by $x=0, x=1, y=0, y=1, z=0, z=1$ (6)

B) Prove that $J_2'(x) = \left(1 - \frac{4}{x^2}\right)J_1(x) + \frac{2}{x}J_0(x)$ (6)

C) Solve $(D^2+3D+2)y = 2(t^2 + t + 1)$, with $y(0)=2$ and $y'(0)=0$ (8)
by using Laplace transform

Q.6) A) Evaluate by Green's theorem for $\int_C (e^{-x} \sin y dx + e^{-x} \cos y dy)$ where C is the rectangle whose vertices are $(0,0), (\pi, 0), (\pi, \pi/2)$ and $(0, \pi/2)$ (6)

B) Show that under the transformation $w = \frac{z-l}{z+l}$, real axis in the z -plane is mapped onto the circle $|w| = 1$ (6)

C) Find Fourier Sine integral representation for $f(x) = \frac{e^{-ax}}{x}$ (8)

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